

~Test 2: Vectors, Circles & Relative(s)

Equations and Constants:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

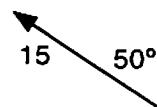
$$\bar{v} = \frac{1}{2}(v_i + v_f)$$

$$|g| = 10 \text{ m/s}^2$$

$$x = \frac{1}{2}at^2 + v_i t + x_i \quad v = at + v_i \quad v_f^2 = v_i^2 + 2a\Delta x \quad a_c = \frac{v^2}{r}$$

Multiple Choice: Choose the letter of the best answer. 3 points each.

1. B What is the vector in unit-vector notation for the vector shown in the diagram to the right?
 a. $-9.6\mathbf{i} + 11.5\mathbf{j}$. b. $-11.5\mathbf{i} + 9.6\mathbf{j}$. c. $-0.77\mathbf{i} + 0.64\mathbf{j}$. d. $-0.64\mathbf{i} + 0.77\mathbf{j}$.
2. A Which of the following has to be true in order to accelerate with a constant speed?
 a. Your acceleration must always be perpendicular to your velocity.
 b. As long as the acceleration is constant, your speed won't change.
 c. Only if your acceleration decreases at a constant rate.
 d. Trick question! This can't be done.
3. B You are flying your plane on a beautiful sunny day when you notice a second plane with a velocity relative to you of $75\mathbf{i} + 50\mathbf{j}$. What is your velocity with respect to the second plane?
 a. $50\mathbf{i} + 75\mathbf{j}$. b. $-75\mathbf{i} - 50\mathbf{j}$. c. $75\mathbf{i} - 50\mathbf{j}$. d. $-50\mathbf{i} - 75\mathbf{j}$. e. $-50\mathbf{i} + 75\mathbf{j}$.

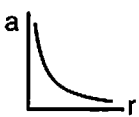



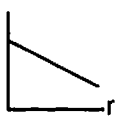


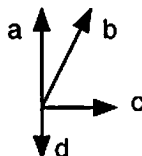
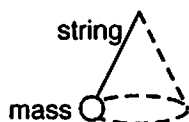
Questions 4 to 6 refer to the following vectors:

$$\mathbf{A} = 7\mathbf{i} + 12\mathbf{j}$$

$$\mathbf{B} = -5\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{C} = 4\mathbf{i} - 5\mathbf{j}$$

4. C What is $\mathbf{A} - \mathbf{B}$?
 a. $2\mathbf{i} + 15\mathbf{j}$. b. $12\mathbf{i} + 15\mathbf{j}$. c. $12\mathbf{i} + 9\mathbf{j}$. d. $2\mathbf{i} + 9\mathbf{j}$.
5. D What is the direction of \mathbf{A} ?
 a. 1.04° . b. 30.3° . c. 35.7° . d. 59.7° .
6. C Which of the following would be perpendicular to \mathbf{B} ?
 a. $10\mathbf{i} - 6\mathbf{j}$. b. $-10\mathbf{i} - 6\mathbf{j}$. c. $-6\mathbf{i} - 10\mathbf{j}$. d. $-6\mathbf{i} + 10\mathbf{j}$.
7. D Which of the following would best represent a graph of acceleration vs radius for an object that is traveling in circles with a constant period?
 a.  b.  c.  d.  e. 
8. C A conical pendulum is a simply a small mass that is hanging from a string, and then set in motion going around in a horizontal circle. The string sweeps out a cone shape as the mass goes around, shown below left, hence the name. When the small mass is at the position shown, which of the arrows shown below right best represents the acceleration of the mass?



e) none are even close.

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Questions 9 to 11 refer to the following:

A little penguin wishes to cross a river. The river flows to the west with a speed of 3 m/s. For some reason, the penguin can only swim with a water speed of 5 m/s. The river is 60 m across.



9. C What is the least amount of time for the penguin to cross the river?
 a. 7.5 s. b. 10.3 s. c. 12 s. d. 15 s. e. it can't be done.
10. D While in the water, what is the fastest speed the penguin can have, relative to the ground?
 a. 3 m/s. b. 4 m/s. c. 5 m/s. d. 8 m/s. e. none of those.
11. D In what direction should the penguin swim relative to the water to land on the opposite side directly across from its starting point?
 a. 0° . b. 31° . c. 37° . d. 53° . e. 90° .
12. A An object has an initial and final velocity as shown in the diagram to the right. The velocity changed over some time interval Δt . Which of the following vectors would best represent the average acceleration of the object?
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Questions 13 and 14 refer to the following:

A group in New York is proposing to build the world's largest Ferris Wheel on Staten Island (which they will modestly call the New York Wheel.) Let's assume the radius of the wheel is 100 meters and the cars moved with a constant speed of 5 m/s.

13. B What would be acceleration of a rider when they were at the maximum height on the ride?
 a. 0 m/s^2 . b. 0.25 m/s^2 . c. 9.75 m/s^2 . d. 10.25 m/s^2 .
14. A With how many rpm would the wheel rotate?
 a. 0.48 rpm. b. 0.75 rpm. c. 1.3 rpm. d. 2.1 rpm.
15. B A car is accelerating at constant speed around a circle with a constant radius. If its acceleration ~~where~~ suddenly doubled, which of the following could be true?
 a. Its speed and radius could have stayed the same.
 b. Its speed could be the same and its radius halved.
 c. Its speed could have doubled and its radius be the same.
 d. Its speed could be four times bigger and its radius doubled.
 e. Its speed could be the same and its radius four times larger.

$$15 @ 3 = 45$$

$$4 @ 10 = 40$$

$$\underline{85 \text{ total}}$$

nick +3
 new jay +3

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~Test 2: Vectors, Circles & Relative(s)**Problem Solving: Show all work. 10 points each.**

16. An object, starting from the origin, has an initial velocity of $15\mathbf{i} - 8\mathbf{j}$ m/s. It then has a constant acceleration of $-2.5\mathbf{i} + 2\mathbf{j}$ m/s². Where is the object when it is at its minimum y-coordinate?

$$0 = 2t - 8$$

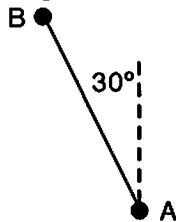
$$t = 4$$

$$x = 15(4) + \frac{1}{2}(-2.5)(4)^2 = 60 - 20 = 40$$

$$y = -8(4) + \frac{1}{2}(2)(4)^2 = -32 + 16 = -16$$

$$\boxed{\vec{r} = 40\hat{i} - 16\hat{j}}$$

17. A plane flies straight from town A to town B in 1.5 hours. B is located 200 km at 30° W of N away from A. There was a crazy wind with a velocity of 50 km/h SW. What was the velocity of the plane with respect to the air?



$$\frac{200}{1.5} = 133 \text{ km/h}$$

$$V_{PB} = -66.7\hat{i} + 115.5\hat{j}$$

$$V_{AB} = -35.4\hat{i} - 35.4\hat{j}$$

$$V^2 = 133^2 + 50^2 - 2(50)(133)\cos(105) = 23,631$$

$$V = 153.7$$

$$\frac{\sin \theta}{50} = \frac{\sin 105}{153.7}$$

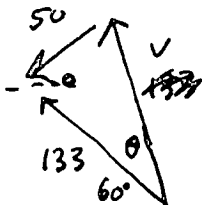
$$\sin \theta = .314$$

$$\theta = 18.3^\circ$$

$$\boxed{\vec{V} = 153.7 @ 78.3^\circ \text{ N of W}}$$

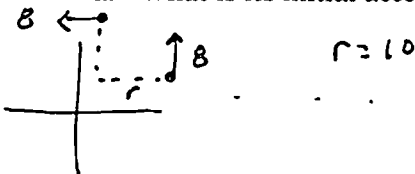
$$\vec{V} = -31.3\hat{i} + 150.9\hat{j}$$

$$= 45 + 60 = 105^\circ$$



18. An object is traveling in a circle with a constant speed. When it is at $25\mathbf{i} + 15\mathbf{j}$ its velocity is $8\mathbf{j}$ m/s. When it is at $15\mathbf{i} + 25\mathbf{j}$ its velocity is $-8\mathbf{i}$ m/s.

a. What is its initial acceleration? (When it is at $25\mathbf{i} + 15\mathbf{j}$)



$$a = \frac{v^2}{r} = \frac{(8)^2}{10} = 6.4$$

$$\boxed{\vec{a} = -6.4\hat{i} \text{ m/s}^2}$$

b. What is its average acceleration over this interval?

$$\bar{a} = \frac{v_f - v_i}{t}$$

$$8 = \frac{1}{4}(2\pi)(10)$$

$$T = 1.96$$

$$\frac{-8\hat{i} - 8\hat{j}}{1.96}$$

$$\boxed{= -4.1\hat{i} - 4.1\hat{j}}$$

$$\boxed{\vec{a} = -6.1\hat{i} - 6.1\hat{j}}$$